

Addis Ababa Science and Technology University



Chapter 4

Structures

Engineering Mechanics I

Content

4. ANALYSIS OF STRUCTURES

4.1. Introduction

4.2. Trusses

4.2.1. Plane Trusses

- Method of Joints
- Method of Sections

4.2.2. Space Trusses

4.3. Pin-ended Multi-Force Structures

4.3.1. Frames

4.3.2. Simple Machines



4.1 Introduction

- This chapter focus on determination of the **force internal to the structure**.
- An **engineering structure** is any connected system of members built to **support** or **transfer forces** and to safely withstand the loads applied to it.
- Internal forces acting on trusses, frames and machines will be analyzed.



4.2 Trusses

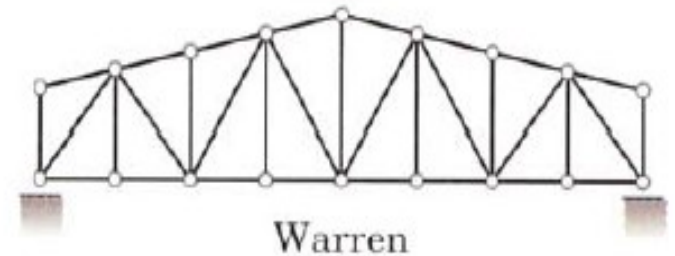
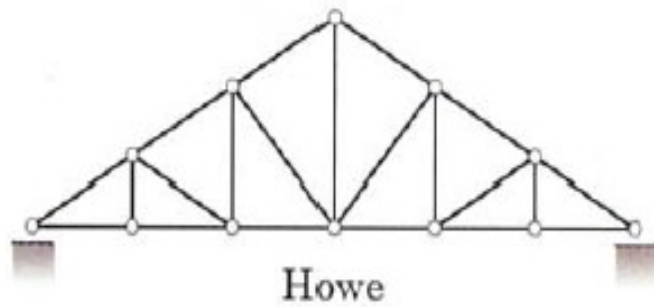
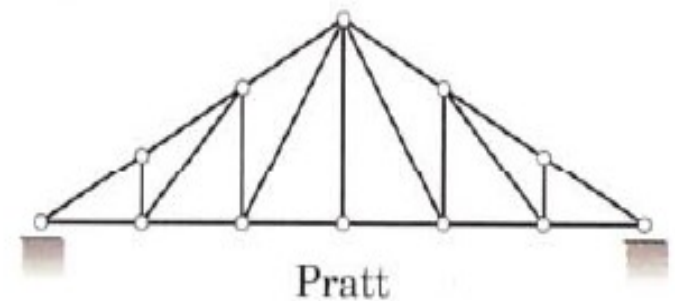
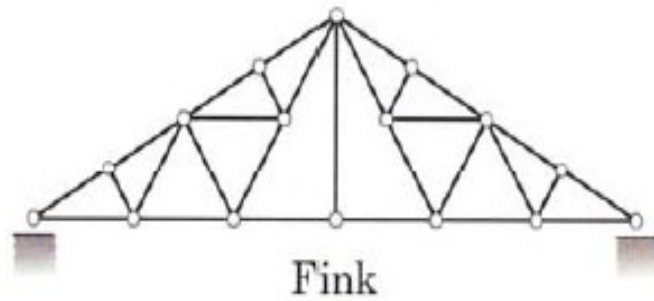
4.2.1. Plane Trusses

- A **truss is a framework** composed of members joined at their ends **to form a rigid** structure.
- When the **members** of the truss lie essentially **in a single plane**, the truss is known as **a plane truss**
- **Bridges, roof supports, derricks** are common examples of trusses





Roof Truss



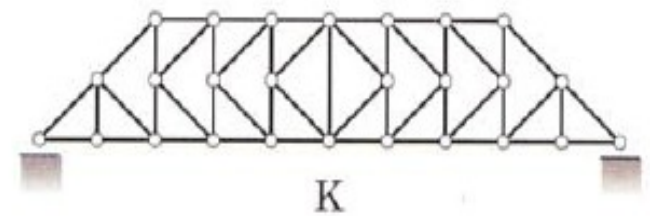
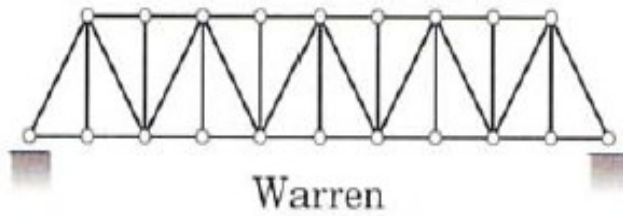
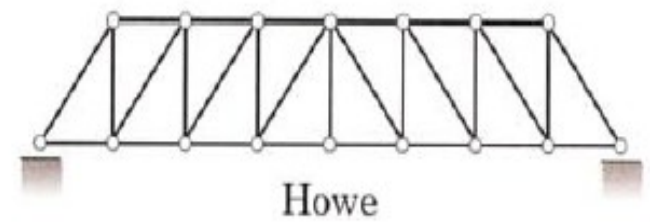
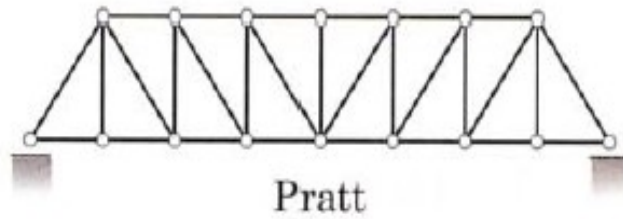
Commonly Used Roof Trusses

Commonly Used Roof Trusses





Tokyo Gate Bridge, a 2.6km long truss bridge



Commonly Used Bridge Trusses

Commonly Used Bridge Trusses





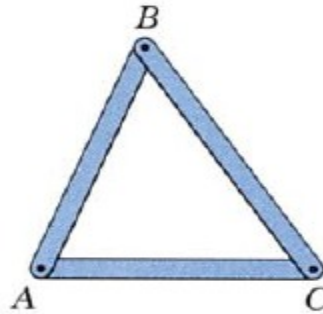
Derricks



Transmission
Towers

Simple Truss

- The **basic element** of a plane truss is **the triangle**.
- Three bars joined by a pin at their end forms a rigid frame



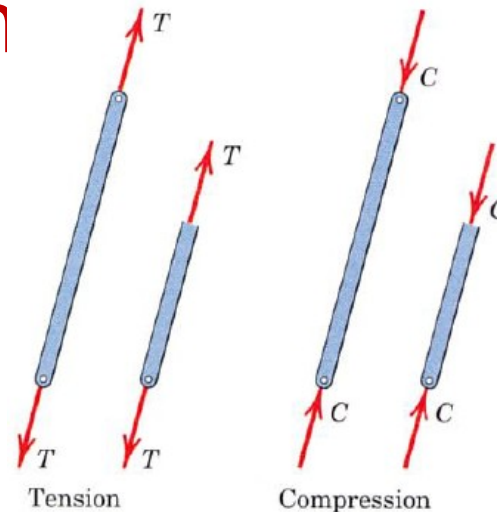
- Structures built from a basic triangle are known as **simple truss**.



Simple Truss

Assumptions in the force analysis of simple trusses

- All members are assumed to be **two-force members**.



- The two forces are applied at the ends of the member.
- The forces are necessarily **equal, opposite** and **collinear**.



Simple Truss

Assumptions in the force analysis of simple trusses

- The members may be in **tension or compression**.
- The tension or compression acting on the cut section is same for all sections.
- If weight must be taken to account, replace the weight W of the member by two forces each $W/2$, acting at each end
- **Connection is pin joint**
- All **external forces** are applied at the pin connections.



Force analysis of plane trusses

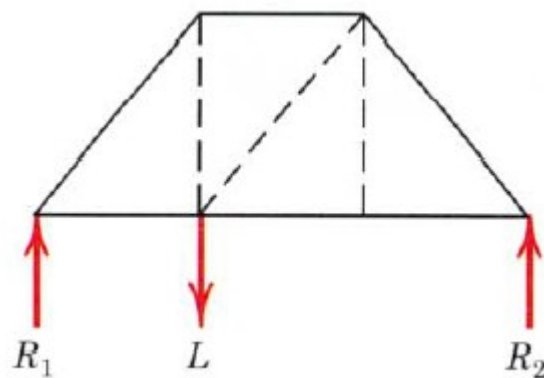
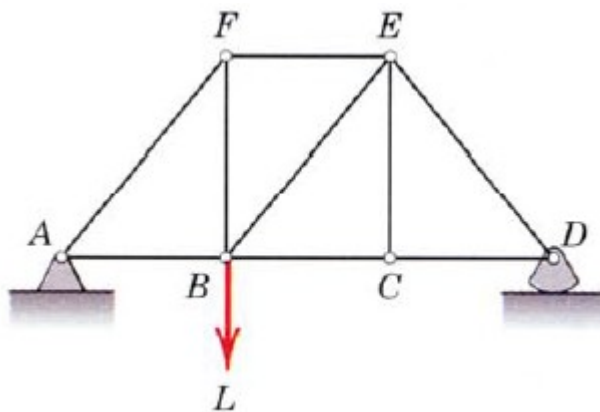
- Two methods for the force analysis of simple trusses will be given.
- The external reactions are usually determined first from the equilibrium equations applied to the truss as a whole before the force analysis of the remainder of the truss.



Method Of Joint

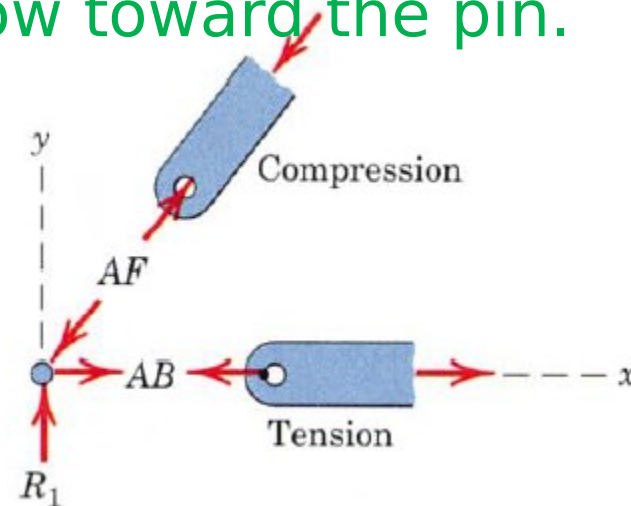
- This method consists of **satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.**
- The method deals with the equilibrium of **concurrent forces**, and hence only two independent equilibrium equations are involved
- We begin the analysis with any joint where at least one known load exists and where not more than two unknown forces are present





External
Reactions are
calculated first

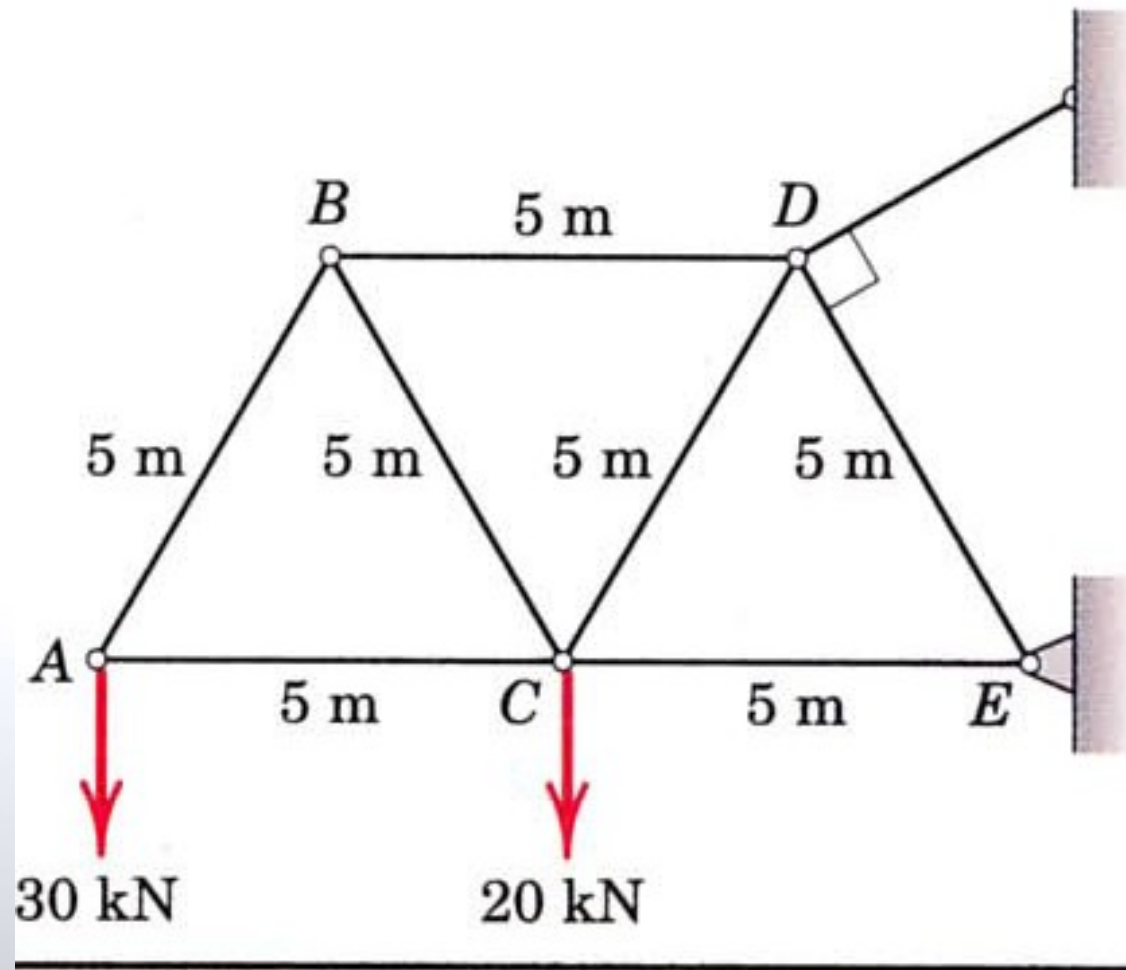
- Taking free body diagram of a joint, **tension** will always be indicated by an arrow away from the pin, and **compression** will always be indicated by an arrow toward the pin.



Equilibrium conditions
must be satisfied
at each joint

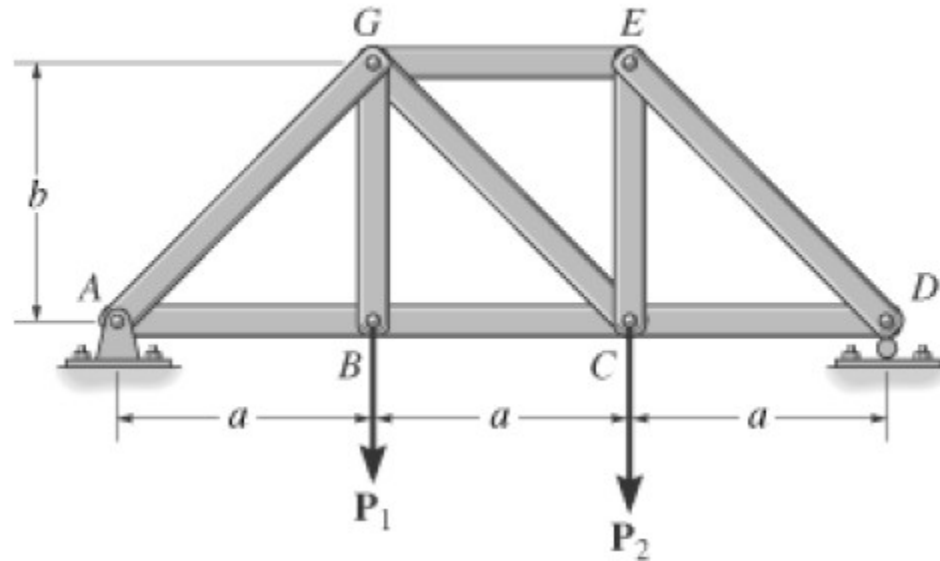


Compute the force in each member of the loaded cantilever truss by the method of joints.



Example

Determine the force in each member of the truss. If $P_1 = 0\text{KN}$ and $P_2 = 1000\text{KN}$ and $a = b = 10\text{m}$

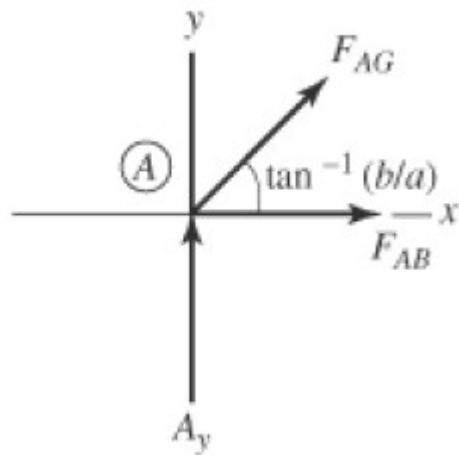


Solution: Determine the external reactions



Solution

Equilibrium equation at joint A



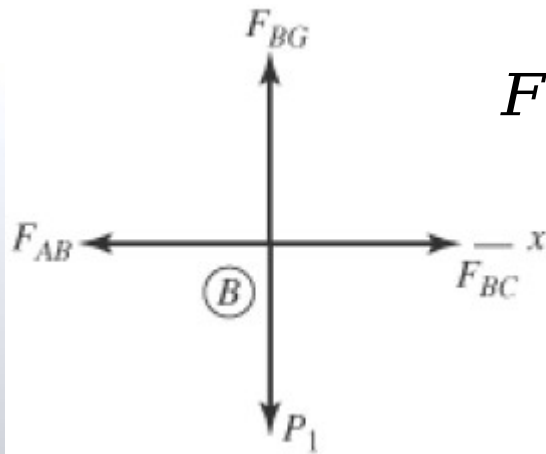
$$\sum F_y = 0$$

$$A_y + F_{AG} \sin 45 = 0 \quad F_{AG} = -471 \text{ KN}$$

$$\sum F_x = 0$$

$$F_{AB} + F_{AG} \cos 45 = 0 \quad F_{AB} = 333 \text{ KN}$$

Equilibrium equation at joint B



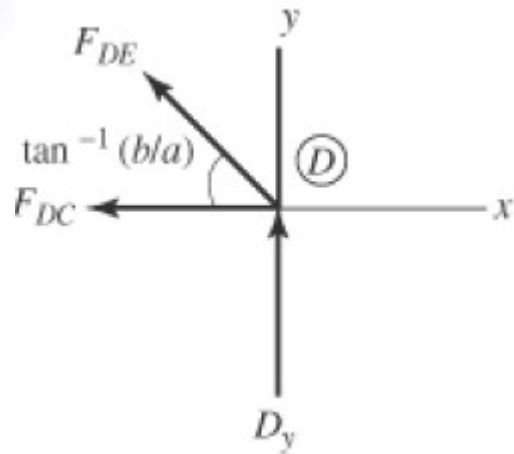
$$F_{AB} = F_{BC} = 333 \text{ KN}$$

$$F_{BG} = 0 \text{ KN}$$



Solution

Equilibrium equation at joint D



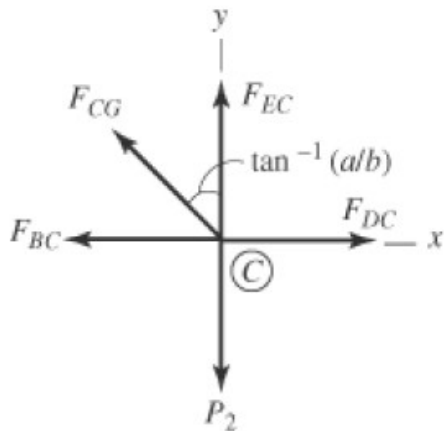
$$\sum F_y = 0 \quad D_y + F_{DE} \sin 45 = 0$$

$$F_{DE} = 667 \text{ KN}$$

$$\sum F_x = 0$$

$$F_{DC} + F_{DE} \cos 45 = 0 \quad F_{DC} = -943 \text{ KN}$$

Equilibrium equation at joint C



$$\sum F_x = 0 \quad F_{DC} - F_{BC} - F_{BC} \cos 45 = 0$$

$$F_{CG} = 471 \text{ KN}$$

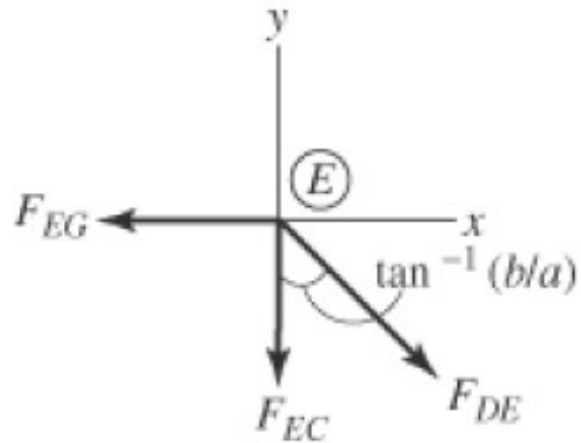
$$\sum F_y = 0 \quad F_{EC} - P_2 + F_{CG} \sin 45 = 0$$

$$F_{EC} = 667 \text{ KN}$$



Solution

Equilibrium equation at joint E



$$\sum F_x = 0$$

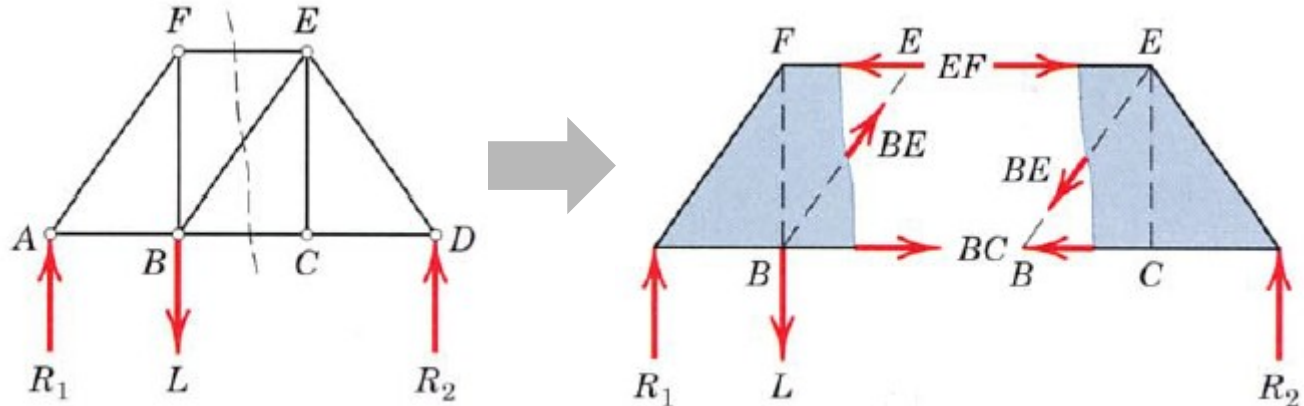
$$F_{DE} \cos 45 - F_{EG} = 0$$

$$F_{EG} = -667 \text{ KN}$$



Method of Section

- Takes advantages of the **third or moment equation of equilibrium**.
- This method has the basic advantage that the force in almost **any desired member may be found directly from an analysis of a section** which has cut that member
- **Not more than three members whose forces are unknown may be cut**



Method of Section

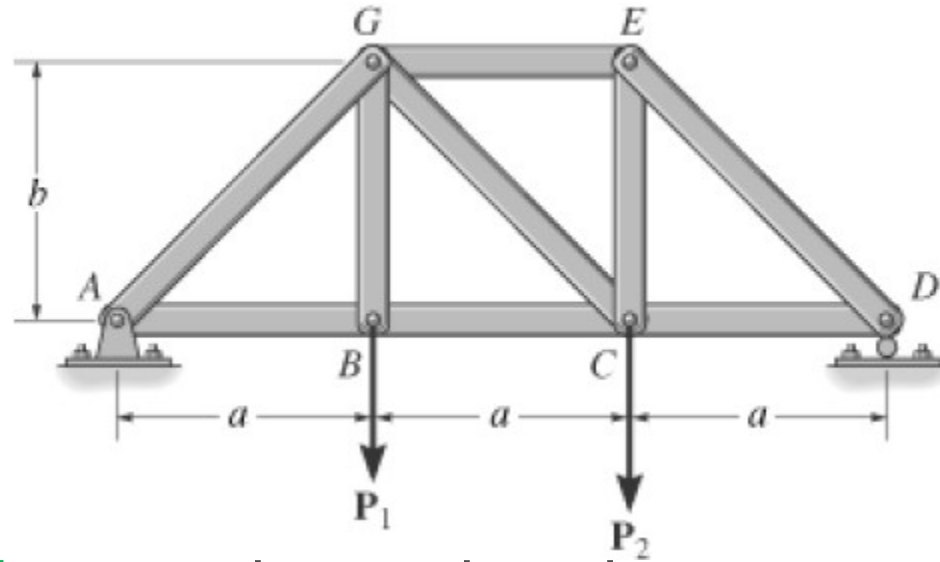
Important Considerations

- Entire portion of the truss is considered a **single body in equilibrium**. Thus, the forces in members internal to the section are not involved in the analysis.
- The **section is preferably passed through the members** and not the joints.
- Methods of sections and joints can be combined for an efficient solution.
- The moment equations are used to great advantage in the method of sections. One should choose a moment center, either on or off the



Example

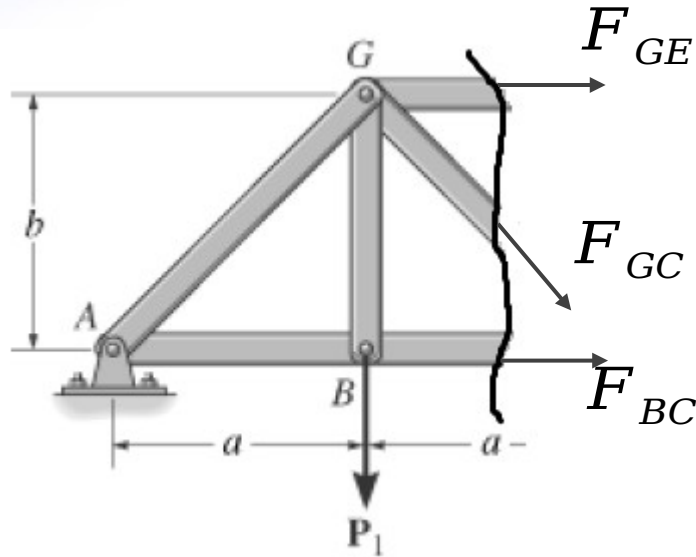
Determine the force in member GC of the truss. If $P_1 = 0\text{KN}$ and $P_2 = 1000\text{KN}$ and $a = b = 10\text{m}$



Solution: 1st determine the external reactions



Solution



$$\sum M_G = 0$$

$$-A_y(10) + F_{BC}(10) = 0$$

$$333(10) - F_{BC}(10) = 0$$

$$F_{BC} = 333 \text{ KN}$$

$$\sum M_E = 0$$

$$A_y(20) - F_{GC} \sin 45(10) - F_{BC}(10) = 0$$

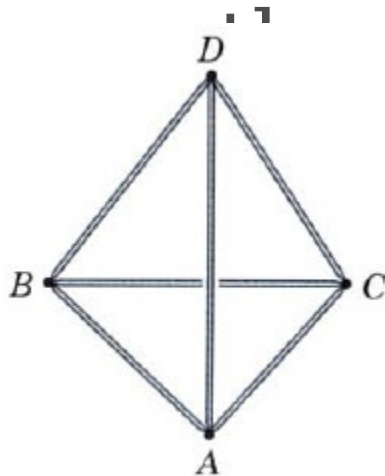
$$333(20) - F_{GC} \sin 45(10) - 333(10) = 0$$

$$F_{GC} = 471 \text{ KN}$$



4.2.2 Space Truss

- A **space truss** is the **three dimensional** counterpart of plane truss.
- Space truss require **six bars** joined at their end to form the edges of tetrahedron basic noncollapsing



- A space truss formed by

Method Of Joint

- This method developed for plane trusses may be extended directly to space trusses.
- Utilizes only the three force equilibrium equations

Method Of Sections

- This method may also be extended directly to space trusses.
- Utilizes all the equilibrium equations



4.3 Frames and Machines

- A structure is called a **frame or machine** if at least one of its individual members is a **multiforce member**.
- A **multiforce member** is defined as one with **three or more forces** acting on it or one with two or more forces and one or more couples acting on it.
- Because frames and machines contain multiforce members, the forces in these members in general



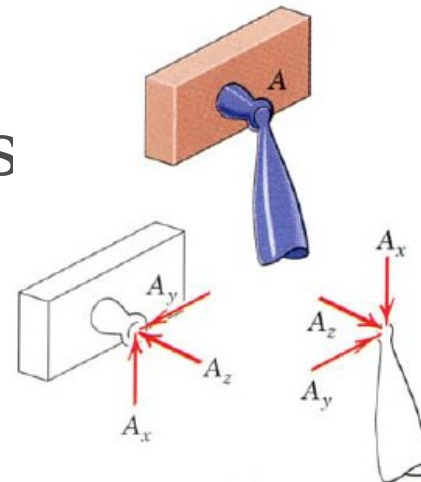
4.3 Frames and Machines

- **Frames** are structures which are designed to support applied loads and are usually fixed in position.
- **Machines** are structures which contain moving parts and are designed to transmit forces or couples from input values to output values.
- This chapter unlike the previous chapter, which deals with equilibrium of single rigid body, concentrate on **equilibrium of**



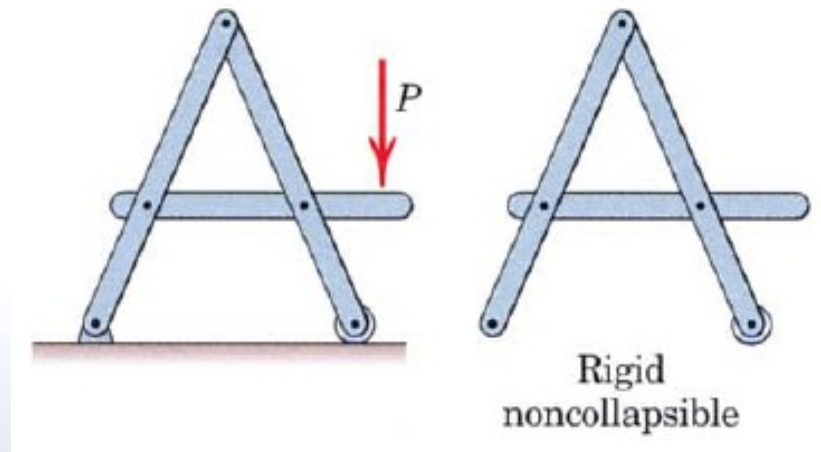
Frames and Machines

- The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the established equations of equilibrium.
- The principle of action and reaction must be carefully observed when we represent the forces of interaction on body diagrams



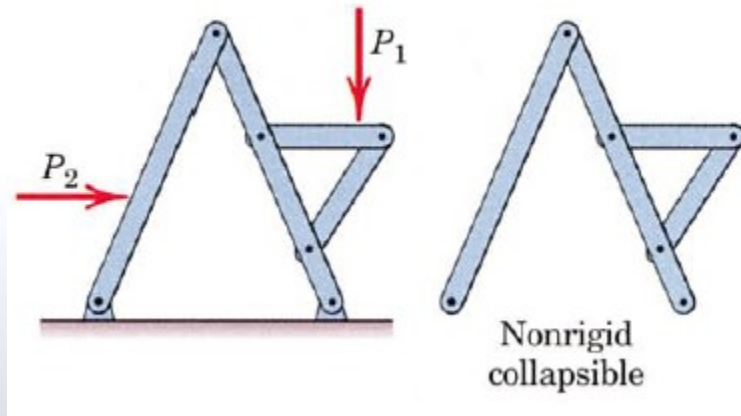
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- If the frame or machine constitutes a rigid unit by itself when removed from its supports, the analysis is best begun by establishing all the forces external to the structure considered as a single rigid body



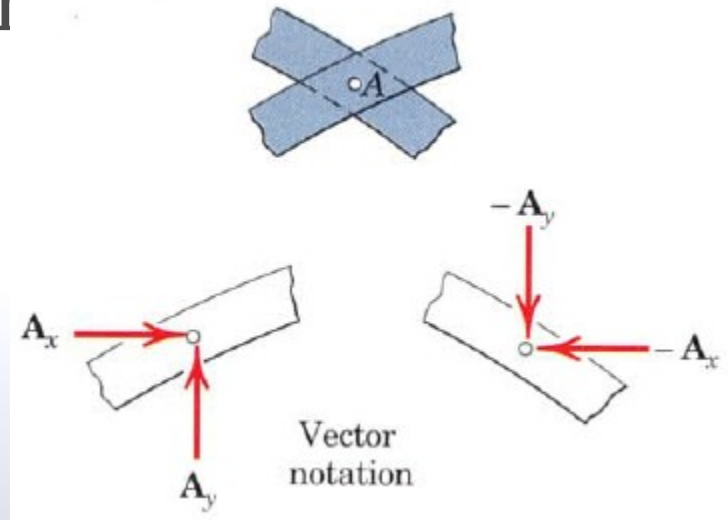
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- If the structure is **not a rigid unit by itself** but depends on its external supports for rigidity, as in the figure below, then **the calculation of the external support reactions cannot be completed until the structure is dismembered** and the individual parts are analyzed



Cont..

- It is **not always possible** to assign every force or its components in the **proper sense**
- In any event it is absolutely necessary that a force be consistently represented on the diagram of the bodies,



THANK YOU!!!

